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What Is Coming in Elementary Mathematics

New experimentation may change mathematics teaching.

WE are perhaps the first generation in history who must educate children for a society which we can only vaguely foresee. We have a tremendous problem in trying to teach a child what he will need to know when he emerges from school 12 to 16 years from now. The complexity of our task might be indicated if we were to try, for example, to make a list of the occupations, industries, government programs, and countries which have come into existence during the past 12 years. Education for the most rapidly changing society in history is useless unless it prepares a child to meet problems that he has never encountered before. The doctrine that the curriculum should be determined by what adults find socially useful is woefully inadequate for our society.

Why Change Is Needed

We must conceive of education as a lifelong process. The main emphasis in the curriculum must be on the tools and the motivation for continued learning, and on the moral and esthetic values which guide our use of knowledge.

Mathematics has three main functions in the curriculum:

1. Deductive reasoning, which is the real subject matter of mathematics as a science, is one of the two methods for acquiring new knowledge at firsthand. (The other is the method of inductive reasoning, the method of the natural and social sciences.)

2. The literature of most branches of knowledge is, to a rapidly increasing extent, written in mathematical language. No matter what vocation a person pursues, he will need to be literate in mathematics.

3. Mathematics is the art of creating beautiful combinations of ideas. In preparing a child for the good life, we must expose him to the arts in all their variety, so that he may learn to appreciate beauty in the work of others, perhaps even to create beauty himself.

Thus a mathematics program, to fulfill its purposes, must give adequate attention to mathematics as a science, a language, and an art.

In particular, the skills of arithmetical computation are an important part of the elementary curriculum, but are far from enough in and of themselves.

Another source of change is the rapid growth of the science of mathematics.

Research does not merely extend the frontiers of our knowledge. It also probes deeply into the foundations of such knowledge. Research often has implications for the most elementary things we teach. Thus the concept of set, which was developed by the work of Cantor about 80 years ago, was immediately recognized as the basis for the number concept. We are just beginning to introduce this concept into the secondary schools. Ultimately this concept will be taught in kindergarten.

Changes To Come

It is too early to predict the details of the curriculum which will emerge as a result of the experimentation now in progress. We can, however, confidently foresee some new emphases, of which the following are examples:

1. *Reasoning.* Children will learn skills of reasoning as well as computation. They may discover the proof, for example, that the product of two consecutive whole numbers is always even. They will learn to apply deduction also to nonmathematical contexts. It is too early to say whether, or how much, formal logic will come into the elementary school. Informal deduction, without a full apparatus of postulates, definitions, and theorems, will certainly come.

2. *Laws of the number system.* The algorithms will be explained on the basis of the underlying laws of numbers, such as:

If a and b are numbers, then $a + b = b + a$ and $a \cdot b = b \cdot a$.

Examples: $3 + 4 = 4 + 3$, $3 \cdot 4 = 4 \cdot 3$.

If a, b, and c are numbers, then $(a + b) + c = a + (b + c)$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Examples: $(10 + 2) + 5 = 10 + (2 + 5)$, $(3 \cdot 2) \cdot 10 = 3 \cdot (2 \cdot 10)$.

If a, b, and c are numbers, then $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.

Examples: $3 \cdot (20 + 1) = (3 \cdot 20) + (3 \cdot 1)$, $(10 \cdot 3) + (10 \cdot 4) = 10 \cdot (3 + 4)$.

Thus the processes for computing $12 + 5$, $3 \cdot 21$, and $30 + 40$ will be explained by means of these laws. Presumably when the children understand the reasons behind the algorithms, they will learn them better and remember them longer than when they learn these processes mechanically as rote skills.

3. *Elementary number theory.* A new intrinsic motivation will be introduced into arithmetic by leading children to discover interesting relationships between numbers. We shall have such problems as:

Calculate each of the products

$$1 \times 2 \times 3,\ 2 \times 3 \times 4,\ 3 \times 4 \times 5,\ 4 \times 5 \times 6, \text{ etc.}$$

and divide your answers by 6; what do you notice about the remainders?

In the present curriculum we motivate learning of arithmetic by showing that it is good for other things, which the children supposedly find interesting. In the future we shall also help children discover that arithmetic is fascinating in itself.

4. *New subject matter.* In the title of this paper we have deliberately referred to "mathematics" rather than "arithmetic." Geometry will certainly be taught from kindergarten up, and not merely in the form of mensuration exercises for practice in arithmetic. It will be primarily the empirical study of the physical space around us, with emphasis

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on observation, experiment, and measurement. A formal deductive development will still be left to the secondary school, but some informal deduction will probably be introduced early. Probably graphs and coordinates will enter rather early in the curriculum.

Similarly, probability as an empirical study of the laws of chance will likely be taught in elementary school. It will be an excellent source of applications of fractions, and will be connected with the study of the biological and social sciences.

Some algebra will be taught early. Subtraction and division will be taught as the processes for solving equations such as

$$2 + x = 5 \quad , \quad 2 \cdot x = 6.$$

Children find the study of equations such as

$$2x + 3 = 11$$

simpler and more exciting than problems such as

$$132\overline{)26938}$$

in the conventional program. Some of the drill in arithmetic will appear in the form:

Solve the equation

$$\frac{x}{15} = 12,$$

rather than:

multiply 15×12.

This seems to work especially well with retarded children.

5. *Integration with other subjects.* We will teach children to describe nature and society in terms of quantitative data. The children will explore the world around them by counting and measuring. They will learn how a scientist predicts and tests predictions, and how we make social decisions on the basis of an analysis of data.

Some of the geometry may appear officially as part of the art curriculum. We shall develop space perception by the construction of drawings and sculptures.

Whatever is written for the mathematics curriculum will contribute to the reading program. We shall consult experts on reading in our revisions of the mathematical curriculum.

We shall thoroughly reconsider the grade placement of topics. While acceleration will not be a primary goal of our reforms, a modest acceleration will be an inevitable by-product. We shall experiment to find ways and means to adjust for differences in ability so that each child may learn at his own pace and depth. Certainly the present widespread practice of a differentiated reading program and a uniform mathematics program will be changed.

We shall certainly take advantage of the psychologists' work on children's thinking. It is scandalous that such basic work as that of Piaget is hardly mentioned in the education of elementary teachers, let alone applied in the design of the curriculum.

Centers of Activity

The School Mathematics Study Group (SMSG) is a national project for the improvement of mathematical education. It is sponsored by the main organizations concerned with American mathematical education, and is financed by the National Science Foundation. Professor E. G. Begle of Yale University is the Executive Director. He has an advisory committee appointed by the presidents of the sponsoring organizations.

SMSG believes that the necessary improvements in this area can be brought about only by effective cooperative efforts. Such efforts must be carried forward by school and college mathematicians, school administrators, and teacher educators working on an equal basis in an atmosphere of good will and mutual respect.

In February 1959, SMSG held a conference on elementary school mathematics. As a result it decided to extend its activities down to kindergarten. In the summer of 1960 an SMSG writing team, working at Stanford University, produced materials for grades 4-6 intended for all pupils. These will be tried out in various parts of the country during 1960-61, and will be revised on the basis of this experience. SMSG may begin work on grades K-3 during 1961 or 1962.

The SMSG elementary panel overlaps in membership with the Elementary School Curriculum Committee of the National Council of Teachers of Mathematics, affiliated with NEA.

SMSG considers that the most urgent task is to find out what substantial improvements can be made quickly on a large scale with our present teaching corps. Its writing team aims to produce materials which can be taught by the teachers now in schools, with in-service education at most during a previous summer or concurrent with the first year of experience with the new materials. Thus the changes introduced by SMSG will be modest and conservative, though they may appear otherwise to some teachers.

The more radical projects described in the following paragraphs are intended to find out what children can learn, and then to educate teachers to teach this properly. As we begin to obtain a supply of teachers who are well prepared mathematically, we shall be able to use such materials on a larger scale.

The University of Illinois arithmetic project, directed by Professor David Page, is at present exploring the use of smaller units of instruction with gifted children in grades 4-6. Sometime later Page may try to develop a systematic curriculum on the basis of his experiments.

Professor Robert Davis, of Syracuse University, has been teaching many new topics to children of all ability levels in grades 3-8. His materials work superbly in the hands of teachers whom he trains to teach. It is not yet clear how well teachers of average ability can learn to teach by the methods he proposes.

The materials by Hawley and Suppes, of Stanford University, on geometry for the primary grades seem to be effective even when taught by teachers of average ability. It will be interesting to see how successful the materials by Suppes on logic for the primary grades will be.

Similar experimentation is being done by Professor Gundlach, of Bowling Green University, under the auspices of the Greater Cleveland Educational Research Council.

Professor F. E. Koehler, of the University of Minnesota, has been writing materials on algebra for the fourth grade which seem to be quite suitable for use by ordinary elementary teachers.

My materials for gifted children in grades 5-6 have been tried out in a preliminary edition in several places, and will soon appear as an ordinary textbook. SMSG is sponsoring a writing project under my direction to produce curricular materials for grades K-3 intended for all children. This will be in conjunction with research by Lydia Muller-Willis and myself on the psychology of elementary mathematical concepts.

As indicated previously, we shall probably see a rather conservative curriculum produced by SMSG for wide use, and several more radical curricula suitable only for well prepared teachers. We shall actually not be able to teach what children can and should learn until we face the problem of the mathematical preparation of elementary teachers.

Problems of Staff

We have about 900,000 elementary teachers, very few of whom have studied any mathematics beyond the eighth grade level. With existing manpower, the task of in-service education of elementary teachers is almost insuperable. While we should certainly encourage elementary teachers to attend the various institutes financed by NSF, it is probably more efficient to concentrate our attention on principals, supervisors, and teacher educators, especially in view of the rapid turnover of elementary staffs.

School systems should be encouraged to use mathematicians from nearby colleges or junior colleges as consultants to their elementary staff. In many cases a well prepared secondary school teacher may be able to assist in the elementary program.

Until the recommendations of such bodies as NCATE that all elementary teachers have academic majors begin to take effect, we are not likely to obtain enough new elementary teachers with adequate mathematical preparation. There will be increasing experimentation in the use of special teachers for mathematics and science. A successful experiment of this kind is directed by J. R. Mayor, Director of Education, American Association for the Advancement of Science. It certainly seems odd that school systems should employ special teachers for art, music, and physical education, and not for academic subjects.

The self-contained classroom may no longer be sacrosanct. There will certainly be experimentation with team teaching and departmentalization in elementary schools.

It is imperative that leaders in curriculum development take an active part in the development of and experimentation with the new materials. The channels of communication between research scholars and school people must be kept open. Each of us must assume his share of responsibility for learning about the new developments and spreading the information.

While we must be aware of the need for careful experimentation and evaluation before new materials are introduced on a wide scale, we must also take our share of the responsibility for trying out the new developments, and cautiously extending their use when we find out what works.

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