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## The Arithmetic Gap

*Tom Loveless and John Coughlan*

**Can U.S. students add, subtract, multiply, and divide better than previous generations? Does it matter?**

In the mid-1990s, U.S. students' mediocre performance on the TIMSS, an international assessment of mathematical ability, captured public attention. But more recently, researchers have pointed to causes for optimism. Scores on the main math test of the National Assessment of Educational Progress (NAEP) have risen steadily in the past decade. From 1990 to 2003, 8th graders picked up 15 scale score points and 4th graders increased their results by a whopping 22 scale score points, which is equal to almost two years of knowledge or 0.69 standard deviation units (National Center for Education Statistics [NCES], 2003).

These data suggest that the United States' future in math achievement looks bright. Should Singaporean kids, at the top of the tables in the most recent TIMSS, be casting nervous glances over their shoulders as the up-and-coming U.S. students rapidly narrow the gap?

## The Trend NAEP

Unfortunately, these encouraging results from the main NAEP mask a significant deficiency in U.S. students' mathematics performance: computation skills. When it comes to students' ability to add, subtract, multiply, and divide using whole numbers, fractions, and decimals and to accurately compute percentages, the picture is not so rosy.

The main NAEP test items are based on the curriculum frameworks developed by the National Assessment Governing Board (NAGB). Because these frameworks are based on curriculum standards developed within the field, the content of the main NAEP evolves to match changing instructional practices. This limits the ability of the main NAEP to measure change reliably over time.

In contrast, the long-term trend NAEP, an entirely different assessment, has used the same testing instruments since its first administration. This stability enables comparisons of student performance over time. In addition, the mathematics trend NAEP has retained a greater emphasis on computation than the evolving main NAEP.

Last given in 1999, the trend NAEP shows more modest overall gains than the main NAEP (see fig. 1). In the 1990s, 9-year-olds gained 2 scale score points; 13-year-olds gained 6 points, and 17-year-olds gained 3 points. The most glaring discrepancy between the two tests is with the youngest students—the 9-year-olds. When the gains are expressed as standard deviations of the 1990 tests—a metric used by statisticians to compare gains on different scales—the main NAEP indicates gains that are more than 10 times larger than gains on the trend NAEP (0.69 compared to 0.06).

| Figure 1. Math Gains of U.S. Students | MAIN NAEP 1990–2003 | | | TREND NAEP 1990–1999 | | |
|---|---|---|---|---|---|---|
| | **Scale** | **Standard** | **Years of** | **Scale** | **Standard** | **Years of** |

| | Score Points | Deviation Units | Knowledge | Score Points | Deviation Units | Knowledge |
|---|---|---|---|---|---|---|
| **Grade 12/Age 17** | 7 | 019* | – | 3 | 0.10 | – |
| **Grade 8/Age 13** | 15 | 0.42 | 1.94 | 6 | 0.19 | 0.69 |
| **Grade 4/Age 9** | 22 | 0.69 | 1.76 | 2 | 0.06 | 0.20 |

*Grade 12 last tested in 2000.

The long-term trend NAEP, with its greater emphasis on computation, shows much more modest gains in the past decade than the main NAEP does. The table shows changes in terms of scale score points, standard deviation units, and years of knowledge.

Source: Author's calculations based on NAEP data from NCES, 2003.

Does the varying emphasis on computation in the two tests explain the contrasting results? To explore this question, we conducted a problem-by-problem analysis of trend NAEP results in the past 20 years.

## Basic Skills Achievement

A wall of jargon confronts interested parents, policymakers, and citizens who want to know whether students can add whole numbers or make sense of fractions. Computation results are hidden in the subcategory of “Number sense, properties, and operations,” which includes problems requiring 4th graders to “choose a number sentence,” 8th graders to “translate words into numbers,” and 12th graders to “apply numerical reasoning” (NAGB, 2001). As valuable as these skills are, we must also determine whether students can compute correct answers after they have chosen number sentences, translated words into numbers, and applied numerical reasoning.

We took trend NAEP results from the past two decades and scrutinized them problem-by-problem to find those that tested students' basic computation skills. We cast aside problems requiring 13-year-olds to “identify an even number” or “identify a number sentence.” To be included in one of our arithmetic clusters, a problem had to test addition, subtraction, multiplication, or division of whole numbers for 9-year-olds and the same operations with whole numbers, fractions, decimals, and percentages for 13- and 17-year-olds—all without a calculator. Given the small number of test items in any one cluster, the data we obtained are not definitive. But some consistent patterns do emerge.

As Figure 2 (p. 58) shows, the results are little cause for celebration. After overall steady increases in the 1980s, the gains slowed, leveled off, and even reversed in the 1990s. Older students' performance on problems involving fractions was particularly disappointing. In 1982, 67 percent of 17-year-olds answered questions dealing with fractions correctly. By 1990, 76 percent answered correctly—considerable progress in less than a decade. By 1999, however, the bottom had fallen out—only 56 percent of 17-year-olds answered correctly. Although this is the most dramatic swing among our skill clusters, evidence of a decline in computation skills abounds. Only in the skill of computing with percentages, which rose for both 13- and 17-year-olds, were there gains in the 1990s.

| Figure 2. Trends in U.S. Students' Computation Skills | Skill Cluster (Items) | 1982 | 1990 | 1999 | Change 1982–1990 | Change 1990–1999 |
|---|---|---|---|---|---|---|
| Age 9 | **Addition (4)** | 71.4 | 75.7 | 75.5 | 4.3 | -0.2 |
| | **Subtraction (3)** | 53.4 | 63.4 | 59.7 | 10.0 | -3.7 |
| | **Multiplication (2)** | 37.4 | 43.9 | 42.5 | 6.5 | -1.4 |
| | **Division (2)** | 44.3 | 49.1 | 48.3 | 4.8 | -0.8 |
| Age 13 | **Whole Numbers (9)** | 92.7 | 91.4 | 89.5 | -1.3 | -1.9 |
| | **Fractions (4)** | 59.7 | 56.8 | 54.3 | -2.9 | -2.5 |
| | **Percentages (6)** | 39.1 | 38.8 | 46.7 | -0.3 | 7.9 |
| Age 17 | **Fractions (3)** | 66.9 | 75.8 | 55.5 | 8.9 | -20.3 |
| | **Decimals (3)** | 51.3 | 50.4 | 46.3 | -0.9 | -4.1 |
| | **Percentages (6)** | 56.3 | 66.5 | 70.0 | 10.2 | 3.5 |

Gains in the percentages of students answering NAEP computation items correctly in the 1980s consistently leveled off or reversed in the 1990s, with the exception of computing with percentages.

Source: Author's calculations based on NAEP data from NCES, 2003.

## Why Does Arithmetic Matter?

Even if students can't compute very well, should we be concerned? We often hear statements reflecting apathy about students' declining arithmetic skills:

> My 9-year-old may not be able to divide 56 by 7 in his head, or figure out that 8 is one-third of 24, but the little guy has his own Web site and programs TiVo for me when I'm away at work. Besides, I was never a "math person," and I turned out OK.

Concern over the recent decline in arithmetic skills among U.S. students is not merely the province of a group of cranky mathematicians. Basic skills are important for three reasons.

*Computation skills are necessary to advance in mathematics and the sciences*. Learning mathematics is an incremental process. Eighth graders who cannot do basic arithmetic with ease, who cannot find the right answer quickly and confidently without a calculator, will be hampered in their efforts to learn algebra and geometry in high school. Without some proficiency in algebra, students will have little grasp of calculus, physics, or chemistry and little chance of succeeding in college mathematics and science courses.

*Computation skills are an increasingly important predictor of adult earnings*. Learning basic computation skills is not just for our future brain surgeons and rocket scientists. In Murnane, Willett, and Levy's discussion of their landmark study on cognitive skills as a predictor of future earnings, they observe that

> a high school senior's mastery of skills taught in American schools no later than the 8th grade is an increasingly important determinant of subsequent wages. (1995, p. 264)

*Computation skills promote equity in math achievement*. Declining arithmetic achievement in the United States also raises concern about racial equity. The achievement gap in computation skills between black and white students narrowed in the 1980s but began to widen in the 1990s (Lee, 2002). From 1990 to 1999, for instance, white 9-year-olds' performance on division problems dropped one-tenth of a percentage point, whereas their black counterparts' performance fell by 6 percentage points. Thirteen-year-old white students fell 2.1 percentage points on fractions compared with a drop of 4 percentage points for black students. Most eye-opening is 17-year-olds' tumble in fractions. White students' performance fell nearly 18 percentage points, whereas that of black 17-year-olds decreased by 33.6 percentage points.

## How Did This Happen?

Although these data are the best we have on a national level, they are hardly robust enough to make a declarative statement of causality. However, we suspect three major factors for this decline, all interrelated: the poor preparation of elementary and middle school math teachers; an increasing reliance on calculators in the classroom from a young age; and math “reform” standards and curriculums that gained favor in the early 1990s.

*Poor teacher preparation*. Recent survey data from the TIMSS suggests that U.S. math teachers are less prepared in their subject area than their more successful counterparts abroad: 78 percent of Singaporean students and 89 percent of Flemish Belgian 8th graders have teachers who majored in math, compared with only 41 percent of U.S. 8th graders. U.S. math teachers majored in education more than teachers in any other country (Loveless, 2001a; TIMSS, 2003). Richard Askey (1999), a math professor at the University of Wisconsin, notes the deficiencies in math education courses taken by elementary and middle school teachers. He derides time-wasting workshops and calls for an overhaul of professional development and a greater emphasis on deepening teachers' understanding of elementary mathematics.

*Use of calculators in elementary school classrooms*. On the NAEP, 4th graders who say that they use calculators every day on classwork have significantly lower math scores than students who never use them. The relationship reverses at 8th grade. Of course, correlation is not proof of causality. Generally speaking, research shows neither a positive nor a negative effect of calculator use on students' computation skills (Ellington, 2003), but most of the calculator studies have involved middle and high school students. Very few have focused on the elementary grades or on the long-term impact of calculators on students who are first learning arithmetic. Even less is known about whether calculators help or hinder students who are struggling to catch up with their peers in mathematics.

*Reformist math standards and curriculum*. The National Council of Teachers of Mathematics (NCTM) came up with the *NCTM Standards* in 1989. The 24 drafters of the *Standards* included faculty from teacher education schools and universities, as well as two K-12 teachers—but not a single mathematician. The vague standards suggest that K-4 students should devote more attention to “operation sense” and “cooperative work” while spending less time doing long division and using pencil and paper to compute fractions. An emphasis on using calculators at all ages runs through the entire document (Klein, 2002; Loveless, 1998; NCTM, 1989).

The publication of the standards coincided neatly with then-President George H. W. Bush's call for national standards at a meeting of state governors in 1989. The NCTM guidelines became a model for U.S. national standards. With the support of National Science Foundation grants, curriculums aligned with the NCTM standards spread throughout state departments of education, school districts, and education schools and universities (Loveless, 2003).

The spread of NCTM-based curriculums met considerable resistance from parents, teachers, and mathematicians who believed that the less demanding programs were shortchanging their children and students. After considerable effort on the part of the national organization Mathematically Correct and other like-minded groups of parents and mathematicians, the state of California replaced its NCTM-based state standards with a more rigorous set of expectations for K-12 students. Similar protests took place in Massachusetts. The U.S. Department of Education's 1999 list of 10 recommended textbooks—all created with NCTM standards in mind—prompted an open letter addressed to Secretary of Education Richard Riley decrying the recommendations. More than 200 prominent mathematicians and policymakers signed the letter (Klein, 2002).

Supporters of the NCTM standards often couch the basic debate in terms of “conceptual understanding” and promoting higher-order thinking versus “basic skills” and the monotony of memorizing multiplication tables (Loveless, 2001b). Berkeley mathematician Hung-Hsi Wu lambastes this “bogus dichotomy,” noting that “conceptual advances are invariably built on the bedrock of technique” (1999, p. 14). Would Einstein have come up with the theory of relativity if he couldn't compute with fractions?

## Basic Proficiency for All Students

Critics deride an emphasis on basic skills as a return to the “drill-and-kill” drudgery of the past. We are not advocating a return to the past, but rather a better preparation for the future.

No one wants students to become robotic human calculators who blindly follow the rules of computation to arrive at correct answers. We would simply like all students to learn how to add, subtract, multiply, and divide using whole numbers, fractions, and decimals—and accurately compute percentages—by the end of 8th grade. Only by mastering these skills will students have the opportunity to learn higher-level mathematics. If U.S. students are proficient in basic arithmetic, they and the nation will be much better off.

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**Tom Loveless** is Director of the Brown Center on Education Policy, The Brookings Institution, Washington, DC; **tloveless@brookings.edu**. **John Coughlan** was Senior Research Assistant at the Brown Center on Education Policy; he now resides in Belgium and works for the European Union.

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