Designing Questions to Encourage

When a teacher on the team came by needing something for his students to do on place value, Mrs. Blair gave him the directions and student recording sheet for 100 or Bust (Schielack and Chancellor 1995), a game involving manipulatives and calculators. Later, when Blair asked the teacher how the activity went, he replied that the students liked it but that he was disappointed that it was so short. It had taken only about fifteen minutes for the students to play the game a couple of times. Blair could not hide her confusion because her class had spent more than two fifty-minute periods on this activity. Blair discovered that the time difference could be attributed to her use of questioning during these activities to promote student engagement in mathematical reasoning.

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Blair, along with the authors and other teachers, had participated in a professional development program designed and presented by the authors of this article. The program focused on experiencing and analyzing worthwhile mathematical activities to address the development of number and operation sense in grades 2–4. At the end of each session, the attendees wrote an instructional plan for each activity in which they had participated. The plan was to include questions for *(a)* encouraging students' exploration of the mathematical ideas in the activity, *(b)* guiding the summarizing discussion at the close of the activity, and *(c)* assessing students' learning. This experience revealed that many of the teachers were limited in their ideas about what questions to ask. Typical questions focused on computational answers or procedures, such as, "What did you get for the answer?" "How did you get that?" The teachers' vision of mathematics needed to be expanded to promote their students' mathematical thinking more effectively.

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Learning What Types of Questions to Ask

During the professional development program, we discovered that we needed a common vocabulary to communicate about questions. When trying out sug-

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gested questions for an activity, we needed to talk about the specific purposes of the questions. In the report *Everybody Counts* (National Research Council 1989, 31), mathematics is described as offering—

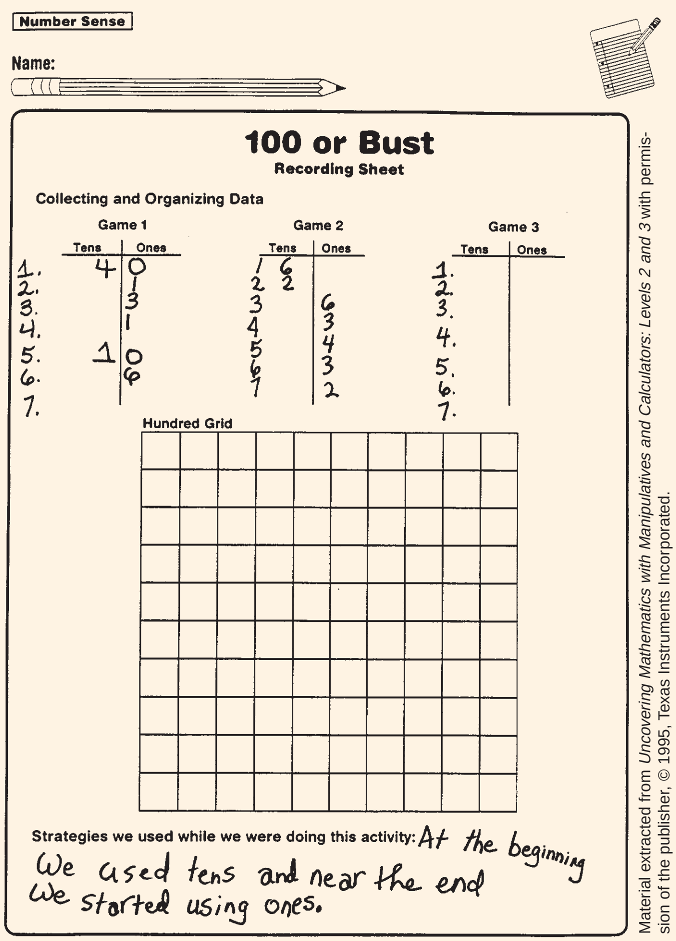
> . . . distinctive modes of thought which are both versatile and powerful, including modeling, abstraction, optimization, logical analysis, inference from data, and use of symbols. Experience with mathematical modes of thought builds mathematical power—a capacity of mind of increasing value in this technological age that enables one to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives.

These modes of thought gave us a way to analyze and discuss the mathematical power of our questions. To have the types of questions fit better into the lesson model that the teachers were using of connecting concrete experiences to abstract ideas, we arranged the mathematical modes of thought into the following categories: modeling (including symbolism), logical analysis, inference, optimizations, and abstraction.

We began by looking at examples of questions for each of the categories in the context of 100 or Bust. In this activity, students roll a die or number cube exactly seven times, each time placing the rolled digit in either the ones or tens place on a recording chart to generate a final sum as close to 100 as possible without going over (see **fig. 1**). The amount chosen for each roll is recorded by placing the appropriate place-value pieces on a hundred-grid, and a running total is kept by entering each

FIGURE 1

Sample of student work for 100 or Bust



addend on a calculator. If students work in groups of three, one student can roll the die and record the selected value on the recording chart, the second student can place the corresponding place-value pieces on the hundred- grid, and the third student can record the outcome using the calculator (see **fig. 2**). Some of the second- and third-grade teachers in the professional development program used this activity in their classrooms, incorporating questions that they developed to promote mathematical reasoning. During the lesson, they recorded students' responses, some of which are shared in the following sections, as evidence of their students' engagement in mathematical thinking.

Modeling

Questions on modeling focus children's attention on visual or symbolic representations that "capture important and useful features" (National Research Council 1989, 3). In 100 or Bust, students construct three different models of the game: the recording chart, the hundred-grid, and the calculator display. Important modeling questions and some examples of students' responses for this game include the following:

Question. What information does the recording chart give you that the hundred-grid and the calculator do not?

Responses. "It helps you know which turn you are on and how many turns you have left." "It helps you go back and see if you could have played your game a different way."

Question. What do the place-value materials on the hundred-grid show that the calculator and recording chart do not?

FIGURE 2

Teamwork can help in this activity.

Responses. "They show you how close you are to going 'bust'!" "It helps you decide whether to make your roll tens or ones by looking at how much you have left to go."

Question. What does the calculator help you do?

Responses. "It keeps track of your total." "You can check it against the blocks on the grid to see if you remembered to punch in the number or not."

Question. What do you do if your models do not match one another?

Response. "You make them match by using the chart to go back over your game step by step."

Logical analysis

Questions involving logical analysis ask students to engage in mathematical reasoning by searching for "first principles to explain observed phenomena" (National Research Council 1989, 3). In this activity, students can analyze the reasons for their decisions for placing each digit. Some logical analysis questions and examples of students' responses for this game include the following:

Question. Why did most people place their first numbers in the tens place?

Responses. "We fill up with lots of tens, then use our ones at the end." "We tried a few games putting our first numbers in the ones place and couldn't get very close to 100 at the end."

Question. How did you decide where to put the small numbers you rolled?

Responses. "Anything we roll that is below a 3 gets to be [in the tens place]." "We chose tens in the beginning if we got 4 or less."

Question. How did you use the information from the models to make your decisions?

Responses. "We are getting close to 100, and we wouldn't want to waste all our spaces." "We already had 10 ones and 3 tens, and we have three turns left." "If we put it in the tens place, then we have to roll 1 on the next roll or go 'bust,' and that's a long shot."

Inference

Logical analysis leads to questions that ask students to make inferences, or engage in "reasoning from data, from premises, from graphs, from incomplete and inconsistent sources to make generalizations and predictions" (National Research Council 1989, 3). From their experiences with and records of the game, students can draw conclusions and investigate other possible outcomes. Some inference questions include the following:

Question. What numbers did you expect to roll? Why?

Responses. "We seem to have a better chance of getting higher numbers." When asked how they knew, "Well, look at the numbers we wrote down! See all the 4s, 5s, and 6s?" [This observation could be followed by having students make an organized list or graph of all the rolls made by the entire class to determine whether rolls of 4, 5, or 6 really did occur more often than rolls of 1, 2, or 3.]

Questions. Can you make up a game that would reach exactly 100 in 7 rolls? How would the game change if you used a ten-sided die with the digits 0–9? How would the game change if you could go over 100, or if you could choose to add or subtract? Could you get closer to 100?

Responses. "We thought we only got to 71, but look! We only took six turns! When we rolled again, we got a 4. If we made it 4 ones, that only gives us 75. But if we made it 4 tens, that would be 111, which is closer to 100 than 75." "If I could subtract, I probably would use a lot more tens. But if we had to get an exact number, like 105, I'd be more careful and use more ones." "I'd go until I busted. Then I'd start subtracting, but then I might need more than seven turns." "If we were in big trouble, we'd subtract. Like if we had 90 and still had lots of turns."

Optimization

One of the most motivating uses of mathematics is "finding the best solution (least expensive or most efficient) by asking 'what if' and exploring all possibilities" (National Research Council 1989, 3). Students can be encouraged to explore and compare ideas of optimal strategies for winning the game by investigating such questions as the following:

Question. What strategies did you develop for getting close to 100?

Responses. "At the beginning, we used tens, and near the end we started using ones." "Don't put big numbers at the beginning. Around the [fifth roll], start using big numbers."

Questions. Why do you think your strategies work? Could you have made exactly 100 with any of your games if you had placed the numbers differently?

Abstraction

Although few individual activities achieve a level of pure abstraction, students can be guided to examine the mathematical principles that are being exhibited in an activity. In 100 or Bust, students can make class graphs of all the numbers rolled to explore which outcomes are equally likely to occur. They can look for patterns in how sums change when a digit is moved from the tens place to the ones place, and vice versa. Students can also explore the relationship between addition and subtraction. Teachers can draw out these mathematical principles with such questions as the following:

Questions. How likely were you to roll the 6 that you needed to make exactly 100? What happens to the sum when you move a 6 from the ones place to the tens place? (The sum increases by 54.) What happens when you move a 6 from the tens place to the ones place? (The sum decreases by 54.) Will this result always happen? Why? How could you change this game so that you use subtraction instead of addition?

Evaluating Activities for Mathematical Reasoning Opportunities

After working through several activities in the professional development program, we developed a list of criteria for evaluating an activity in terms of its potential for engaging students in mathematical reasoning. The teachers used the criteria to critique the activities in the sessions, as well as their own favorite activities, for opportunities to engage students in mathematical reasoning. At this point in the teachers' development of questioning skills, they learned to ask *themselves* some important questions about the activities that they choose for their students.

Opportunities for modeling

How does the activity promote mathematical modeling? What opportunities do students have to make diagrams, pictures, charts, or tables? How does the activity require students to communicate their results through various representations, including appropriate mathematical language and symbolism? The teachers found that they could strengthen many of their favorite activities by allowing students to create and discuss their own representations of the problems rather than present students with a prefabricated model.

Opportunities for logical analysis

How does the activity prompt students to collect, record, and analyze data to discover why things work the way they do? What opportunities do students have to compare and contrast results or notice important patterns to uncover the important characteristics of what is going on in the activity? The teachers found that without an important question to investigate, many activities did not encourage logical analysis. They began to look for activities that were built around developing and analyzing patterns that led to useful conclusions.

Opportunities for making inferences

How does the activity prompt students to collect and record information from which to make generalizations and conjectures? What opportunities do students have to encounter new situations in which to test their conjectures? The teachers began to realize that an important part of mathematical reasoning is determining how new information can be used, and they began to incorporate applications of the ideas learned from an activity.

Opportunities for optimization

How does the activity involve students in trying to find the best or most efficient way to do something or to explore other possibilities through "what if" questions? Both students and teachers found this aspect of mathematical thinking to be the most natural and attractive. Teachers adapted many activities to promote students' development of the most efficient game strategies or least expensive costs.

Opportunities for abstraction

What is the mathematical purpose of the activity? How does it offer students opportunities to form connections and relationships among major ideas or concepts? Using this criterion, the teachers were able to justify the time spent on often lengthy activities, allowing students to deepen their understanding of important mathematical concepts.

Conclusion

By participating in the process of developing common language to discuss the reasons for asking questions and the criteria for evaluating the mathematical reasoning potential of an activity, the teachers expanded their vision of what mathematical thinking is and became more aware of how and when it can occur. In preparing instructional plans for the activities experienced during the sessions, the teachers found that the focus of their questions shifted from answers and procedures to observations and uses of patterns, comparisons of different strategies and representations, and connections among mathematical ideas.

In helping teachers improve the mathematical reasoning of their students, the specific categories of mathematical thinking discussed in this article are not as important as the process of identifying common language for discussing the many aspects of mathematical reasoning, then using that language to create powerful questions. This process encourages teachers to develop criteria for evaluating activities on the basis of the opportunities that the activities afford for asking powerful questions. The goal is for students to continually hear teachers ask questions that explore the use of mathematical models and symbolic representations, that encourage analysis and inference, and that highlight optimal procedures and the construction of mathematical ideas. As a result, when students are faced with a problem, they will begin to create similar questions for themselves and become better mathematical thinkers by independently pursuing applications of modeling, symbolism, analysis, inference, optimization, and mathematical abstraction.

References

National Research Council. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Washington, D.C.: National Academy Press, 1989.

Schielack, Jane F., and Dinah Chancellor. *Uncovering Mathematics with Manipulatives and Calculators: Levels 2 and 3.* Dallas, Tex.: Texas Instruments 1995.

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