Two | Planning Effective Math Instruction

Rhonda felt comfortable with the content of the math curriculum at her grade level, but she felt she needed to improve how she presented it. She used various approaches for planning lessons and had been working on learning to assess what her students knew, incorporating more activities into math instruction, and using cooperative groups. But she still had many questions. "When do cooperative groups make sense and when don't they?" she wondered. "How can I meet the needs of students who finish assignments quickly? What about those kids who always seem confused? What sorts of questions should I ask to best support learning?" Rhonda's concerns are valid. Her search for answers is the key to improving her teaching practice. It's a career-long pursuit, and improvement happens one year at a time.

12 How can I structure my daily math period?

There's no single best way for all math periods to be organized. The organization depends on a myriad of conditions—the materials you plan to use, how you want the students to work, and the assignment you plan to give. Keep in mind that varying the structure of math lessons is a way to maintain interest and keep math classes from becoming humdrum.

That said, it's beneficial to think about the different ways you can organize your class for math learning and your expectations for the students in each situation. For example, there will be times when students will work with a partner or small group and other times when they'll work individually and not share ideas. In the first situation, it's appropriate for students to talk with one another, but that's not so in the second scenario. Also, there are times when students will use particular materials—manipulatives, scissors and glue, specific books, and so on. Again, guidelines should be clear. Whatever organizational structure you use, your goal is to present clear guidelines that the students understand so that they can switch from one setting to another with minimal disruption. Guidelines should address, for example, how to use and put away materials, whether students have to sit in a particular place or can pick a location, whether they will choose a partner or be assigned someone to work with, and so on. Establishing guidelines sets the conventions for working during class that can help lessons go more smoothly and can help you focus more on learning than on managing.

Some teachers like to prepare a binder of information for visitors who come to the class, whether they are parents, teachers from other classrooms, or other observers. The binder contains information about the class's educational program and also about classroom procedures and logistics. Teachers have included a variety of materials in their binders—portions of the district guidelines regarding learning and expectations, lists of topics that will be taught during the year, examples of assignments students will be expected to do, samples of student work, cartoons that communicate a useful or important message, and so on. Creating such a binder with your students is a way to involve them in thinking about their classroom goals and structure. While the binder isn't just for math, it can contain a page titled "How We Work During Math Class." Working on this page with the students can help clarify your expectations. When visitors come, a student host can introduce the binder to them as a way of helping acquaint them with the class.

13 What kinds of questions best support math learning?

Questions are vital for supporting math learning. They invite students to think mathematically, connect what they already know to what they're currently studying, formulate and communicate new ideas, justify procedures, and defend the reasonableness of their answers. Also, students' responses to questions give you information that helps you assess what they know. Students need to understand these two purposes of questions: to help them learn and to help you know what they understand.

With these broad purposes in mind, we think about questions in several categories. One category includes questions that call for one correct answer, such as when you ask children how much is 4 plus 3, 15 plus 23, or the sum of any two numbers. Questions that have a right answer are most effective for promoting mathematical thinking when they have answers a child can figure out, not when they call for some memorized bit of information. There are times, of course, when you ask a question that requires a memorized answer, for example, "What do we call the number on the top of a fraction?" But keep in mind that questions like these focus on mathematical conventions, not mathematical reasoning. The bulk of classroom questions should foster thinking.

Another category of questions are those that call for more than one correct answer. For example, instead of asking for sums as above, ask for two numbers that add up to 7 or 38. There's more than one correct answer to these questions, offering the opportunity for more children to respond. While these questions still focus on addition, they also support thinking about relationships among numbers.

Another category of questions focuses on asking children to explain theories or make conjectures. They may have one correct answer or more than one correct answer, but these questions probe more deeply. Here are examples of questions that fit in this category:

> How many different pairs of addends are there that add to 7? to 38? to any number?
>
> How can you tell without dividing if a number is divisible by 6? by 20?
>
> Why can't a triangle have more than one right angle?
>
> How might you change the rules of a game so that it's fair for both players?

Included in this category are questions that ask students to apply what they've learned in new situations. For example, try asking "What if?" questions, changing one or more conditions of an original problem or situation:

> What if we rolled three dice? What sum will most likely come up more often than others?
>
> What if we wanted to find three numbers that add up to 7?
>
> What if we add two even numbers? Will the answer be even or odd? What if we add two odd numbers or one even and one odd number? What if we multiply?

Avoid questions that can be answered merely by "yes" or "no," because these can close down instead of open up discussion. For example, instead of asking, "Do you understand what ________ said?" ask, "Can you explain why ________'s idea does (or doesn't) make sense?" At every opportunity, questioning should contribute to the development of students' mathematical reasoning. We know that students have caught on to the importance of questions like this when we hear them asking similar questions of one another, often in the same tone of voice we use!

14 I know it's important for students to understand what they're learning. But sometimes I just want to tell them how to do something. Is this all right?

The nature of what you're teaching is the key to deciding when it's appropriate to tell students something and when telling won't serve their learning. What's important here is whether the source of the knowledge for the child is external or internal. We'll try to explain.

Some knowledge is based solely on social convention or customs and, therefore, is accessible only from sources outside the child. For example, there isn't a logical reason that we celebrate Thanksgiving on a Thursday in November each year; it's an arbitrary social custom. So is setting a table with forks on the left of the plate and knives and spoons on the right. There isn't any way to figure out or discover this sort of information by relying only on your internal resources. You have to hear the information, read about it in a book, see it on television, or learn it from some other means. From repeated use, you memorize this kind of information and draw upon it when you need it. If you forget it, you'd have to check with someone else or look it up again; you couldn't figure it out.

In mathematics, we use many social conventions that children have to learn. The plus sign, for example, and the symbols for subtraction, multiplication, and division are all arbitrary conventions. Also, we use commas to separate digits in large numbers and a period to indicate that what follows is a decimal. In England, however, it's the reverse; a comma indicates a decimal numeral and periods are used to separate digits in large numbers! Therefore, teaching by telling is appropriate for helping children learn conventional symbols; you're imparting knowledge that is social in nature and for which the source of the knowledge is outside the learner.

Learning most of mathematics, however, relies on understanding its logical structures. The source of logical understanding is internal, requiring a child to process information, make sense of it, and figure out how to apply it. Hearing and then memorizing doesn't develop understanding. While a child can be told how to read a large number—32,475, for example—and told the names of the places—ones, tens, hundreds, thousands, and so on—understanding the place value system of our numbers requires that they learn its structure. We can explain the logic, but children have to make sense of it for themselves. To engage children in learning the logic of mathematical ideas, we involve them in activities that promote their thinking and reasoning. Working with problem-solving situations helps, using manipulative materials helps, and so does explaining ideas and considering the ideas of others. But the learning goes on inside each child's mind.

All of us have learned the procedure for dividing fractions and can probably get the correct answer to a division of fractions problem. Sadly, however, many of us can't really explain the logic of why we turn one fraction upside down and then multiply. It's as if we were taught: Yours is not to question why, just invert and multiply. The procedure works, but to teach it as if it were a social custom rather than a method steeped in basic principles of numerical understanding doesn't support building mathematical competence. It's a shortsighted decision.

So . . . this is a long-winded way to advise you to let the source of the knowledge you're trying to teach your students determine how you'll teach it. If it's social knowledge, telling is fine. If the knowledge calls for understanding a logical construct, then not only is telling a poor approach, but it leads to mathematical dead ends.

We believe this theory is sound. But we also know that in the thick of classroom instruction, even the most solid theory can be difficult to heed. Many of us, for example, have experienced a student saying to us, "I don't get it. If you'll just tell me what to do, I'll do it." This is usually when a student is more focused on finishing an assignment than on understanding. Or, worse yet, it's when a student thinks that something is just supposed to be memorized rather than to make sense. While the temptation in situations like these may be to relieve the tension by telling students what to do, it's important to remember the benefit of this decision is short-lived.

15 How can I assess my students to find out if they're learning what I'm teaching?

Assessment is an ongoing aspect of instruction, and it's useful to think about assessing your students in several ways. One is by listening to the responses they offer in whole-class lessons. Another is by walking around the classroom when students are working in small groups or individually, listening to what they are saying to one another or asking questions to probe their thinking. Both of these are informal opportunities that occur in conjunction with instruction. The information you gather, although seemingly incidental in nature, is valuable.

For example, many teachers use the day's date to help their children focus on writing equations for numbers. On the ninth day of the month, for example, the children think of number sentences that produce the number 9, and the teacher records their ideas on the board. For young children, this most often initially translates to thinking about addition combinations and later to subtraction problems as well. For older students, the teacher can make other requirements such as having to use each of the four operations at least once ($4 \times 6 \div 8 + 10 - 4 = 9$), having to use only four 4s and any combinations of operations ($4 \div 4 + 4 + 4 = 9$), or using the number 100 in the sentence ($100 - 81 = 9$). An activity like this is not only a good way to begin the day's math class, but it's also useful for listening to children's responses and learning about their numerical comfort and ability.

Instead of having a whole-class discussion to collect equations, however, children can work in pairs or small groups to brainstorm number sentences. In this case, circulate through the room and listen as students work. After a few minutes, call the class to attention. Then go around the room and give each group a chance to offer a number sentence for you to record on the board. Make the rule that a group must report a sentence that you haven't already written. Groups may pass if they have no new ideas, but continue until groups have offered all of their thoughts.

While observing in whole-class and small-group settings provides valuable information about students, it's also important for you to find out specifically what each student knows. Many children are talkative and willing to participate in class. Their hands shoot up in the air regularly to provide answers and explanations. But other children are reticent to volunteer in whole-class discussions, and some are also quiet in small groups. You need to have ways to collect information about these children's learning as well, and you can accomplish this through written assignments or one-on-one interviews.

For example, after listing half a dozen or so of the students' number sentences on the board, ask children to write five more equations on their own. You may want to give an extra challenge to this assignment. Here are some options for extensions:

Think of sentences with three or more numbers that add up to our date.
Write sentences that include at least one multiplication.
Write sentences that use only fractions or decimals, no whole numbers.

Their papers will reveal their capability to do the work; their choice of numbers can give you further information about their numerical comfort.

A one-on-one interview is probably the most valuable way to assess the depth of a child's thinking. Doing so with even a few of your students will not only give you access into those children's understanding but also help you think more deeply about the mathematics you're teaching.

16 I'd like to try some one-on-one interviews to assess my students. How can I do this during class time?

There's really no other way to assess a student's understanding that's comparable to conducting one-on-one interviews. Talking with individual children gives you a window into their thinking and reasoning that simply isn't available from students' written work or participation in whole-class discussions.

There are several aspects to preparing for one-on-one interviews—deciding what questions you'll ask, making time to hold an individual conversation, setting the stage with a student for a successful interview, and making use of the results. Of these, we find the first aspect, deciding on the questions to ask, to be the most critical. Think about sitting across a desk from one of your students. What do you want to know? What questions might you ask? What problems would be suitable to reveal the child's understanding and skills? Remember that your goal is to gain understanding of children's reasoning as well as what they can do.

For example, suppose you want to assess a second grader's numerical facility. You might start by asking the child to count out 20 beans or tiles. Cover them with a sheet of paper and then remove 6 or 7 of them. Show the child the beans or tiles you removed and ask how many are still under the paper. Does the child know instantly? If not, how does the child figure out the answer? Is the answer correct? Can the child explain how he or she reasoned? Remember that an answer without an explanation doesn't reveal how a child thinks. Don't rely on doing this just once. If a child answers incorrectly, try again with a number that's smaller than 20, perhaps 10. If a child answers correctly, pose another problem as a check. Also, you might ask the child to write down a math sentence or equation that describes this math problem.

With older students, you might be interested in their understanding of fractions. Pose a problem that calls for using fractions. It could be something they've done in class so you can see how they work on their own. For example, ask a student to share cookies fairly among a group of friends. Ask, "If there were 4 people and 3 cookies, how much would each get?" Be sure that the student knows that fair shares mean that each person gets the same amount of cookie. As with the younger children, ask the student to write a math equation to describe the problem and also to record the amount of cookie each person gets. Change the number of people and cookies for follow-up problems.

For both younger and older students, keep interviews fairly short. We find that five or ten minutes used well is enormously helpful to learn about a child. And a short interview makes it easier to figure out a time when you can conduct the assessment without being interrupted. It's difficult to do when you're in the midst of managing whole-class instruction. If it's important to have a one-on-one conversation with a particular student, arrange to do so when the other students are elsewhere, perhaps at the library. If an interview is short, it shouldn't pose a sacrifice to have a child miss a particular activity this one time, especially if the information you learn will help you meet that specific child's needs.

About setting the stage: We recommend that you tell the child why you're having the conversation, that you're interested in finding out as much as possible about what he or she does and doesn't yet know in math so that you can think about how to help him or her learn. Tell students the most important thing is that you understand how they think, that you can't peer into their eyes or peek into their ears and get a glimpse of how their brain works mathematically. The only way you can understand how their brain is working is to ask them to tell or show you. That's why you want them to talk out loud as much as possible and, at times, write things down.

You can make use of what you learn in individual interviews in two ways: to gain insights into the individual student's math progress and to inform and improve your overall classroom instruction. Think about the questions you asked that gave you specific information about the student, information that would help you, for example, in a conversation with the student's parents or with another colleague about the student's math progress. Then think how to integrate those questions into your classroom teaching.

We realize that it's difficult to arrange and manage individual assessments in light of the demands of a typical teaching day, and it's most likely not possible to arrange for one-on-one conversations with all students. But we urge you to pursue individual assessments with at least some of your students. We've found the benefits to be well worth the effort.

17 What should I do if I plan lessons that are too hard or too easy?

Without a doubt, your students will let you know if something you try with them is too difficult. The feedback may be in the form of a barrage of questions and comments relating to the assignment: "What do I do?" "I don't get it!" "Can you help me?" Or, the remarks may be less related to the lesson you've presented or the assignment you've given and appear to be a negative attitude: "This is boring." "I don't like this." "Do I have to do this?" Also, children's responses in a whole-class discussion or on their written work can let you know when they don't understand.

However you get the feedback, the best way to deal with it is to acknowledge it. Say to students, "I don't think what I planned is helping you learn. It seems like you're confused and frustrated, and I'm not feeling good about the lesson. What can you tell me about the problem?" Then listen. Accept what they say without being defensive. As students verbalize what they don't understand, try to ascertain whether they have all of the skills needed to acquire understanding. For example, young students facing the addition of numbers in the teens may not yet be comfortable enough with number combinations for 10. Students have difficulty estimating the product of two- and three-digit numbers if they don't understand the patterns of multiplying by 10, 100, and 1,000. Remind students that it's your job to help them learn. Make sure you explain that the math ideas are important for them to know, and that you'll try another approach that might be less confusing and more successful. Your new approach may involve a more concrete, hands-on experience or a way for them to review previously presented material. In any case, your children need to know that you think they're capable of learning the material and that you'll take responsibility for presenting it in a way that they can be successful.

You'll also be able to sense if something you present is too easy. Sometimes students will whip through an assignment in much less time than you anticipate. Most often, however, an assignment is too easy for only a handful of students who may remark, "We did this last year." When you plan lessons, have extensions in mind for these students. The purpose of the extensions isn't to load them down with more work, but to extend the lesson—to challenge them with a thinking activity that will engage them. (*See page 28 for ideas about extensions.*) When introducing an extension to children, you might say, "I notice that the work I've assigned is really easy for you. That's terrific. But what's important in math is that you can think about something you know in different ways. Here's something I'd like you to try. I'm interested in seeing how you make sense of it. And I'm interested in your opinion about whether you think other students would benefit from this new challenge."

It's not unusual for some students to find the work too hard or too easy. The best preparation for dealing with these situations is to know whether students possess the prerequisite learning for the lesson as well as how to extend the mathematics. Your intent should be on giving children the opportunity to work to the edges of their own ability boundaries.

18 I've heard that when assigning a problem, first you should have volunteers discuss how they might go about figuring it out. Doesn't this give too much away for the others?

It helps to make a distinction between a learning activity and a testing situation. When you are testing each child's ability to solve a particular kind of problem or demonstrate a particular skill, then it makes sense to ask the children to work independently in order to show what they can do. But the bulk of the children's math time should focus on their learning, which means providing them the most help you can in order for them to understand something new or build new learning on top of what they already know. A class discussion helps make different approaches and ideas accessible to others. It can help a child who was perplexed know how to begin. It can offer alternative approaches to students who already had a particular idea. In either case, nothing is lost, and increased learning can result.

Sometimes in an initial discussion, a volunteer blurts out an answer. When this happens, reinforce that you're interested both in the answer and the thinking they did to figure it out. One way to respond is, "I'm not interested in just the answer at this time. I'm interested in your ideas about how you might go about finding the answer or checking that an answer you have is correct."

Then ask each child to record a solution and explain how he or she reached it. It's fine for students to use an idea suggested by a classmate, but they should explain it in their own words. Keep the learning focus clear: *Do only what makes sense to you.* Also, make a practice of asking students to think about at least two different ways to figure out an answer. This encourages students to try out others' ideas and become flexible in their mathematical thinking.

19 What about cooperative groups in math class—do they really help?

We think that having students work cooperatively is extremely helpful. In order to make sense of the math they're learning, students need as many opportunities as possible to talk about what they're thinking and how they're reasoning. Cooperative groups can make that happen. When working in groups, students talk about their ideas and learn from others' thinking, which helps them develop, cement, and extend their understanding.

Of course, merely putting students in groups doesn't guarantee learning. First of all, you'll need to work with students so that they have clear guidelines for being a productive member of a cooperative group, whether it be a pair of children or a group of three, four, or five students. An important expectation is that all of the students in a group participate and understand what the group is doing. Also, you should be able to question any group member at any time and get a report on the group's progress.

There are different ways that groups can work. At times they'll work together, collaborating on an investigation or solving a particular problem. One useful rule of thumb when you want the group to work closely together is to let the students know that you'll offer your assistance when everyone in the group has the same question, and only then. This helps avoid a situation in which one child has a question and seeks your help before talking with the others in the group. At other times, students will be doing individual work, but their group members will be available to support, willing to help when asked. And at still other times, you'll want students to work individually without help from classmates. This is essential for finding out what individual students know. In these cases, give the children guidelines about what you expect. When students are given clear information about how you want them to work and why, they're more apt to understand what you want and be willing to conform.

Teachers have different systems for putting children into groups. Some have children sit in groups all of the time so that it's easy for them to work together when directed to do so. While some of these teachers leave the groups more or less the same for the year, others change the seatings on a regular basis, such as every month. Many teachers place students into groups randomly; one method is to distribute playing cards and have the aces, twos, threes, and so on sit together, using two, three, or four of each number card, depending on the size groups you're forming. Other teachers prefer to assign partners or groups. Whatever the system, both the mechanics and the rationale for your particular method should be clear to the students. For example, a reason for grouping children randomly might be because you feel it's important that they have the opportunity to work with all of their classmates during the year. Or you might assign partners or groups because of children's particular interests.

When first introducing students to working cooperatively, it's important to choose math activities that foster cooperation. Be sure the task is appropriate for all students, choose a task that's accessible to those with less ability while also offering a challenge to students with more aptitude and interest. For example, second graders worked in pairs and compared the first letters of their names. Together they wrote two lists, one describing how the letters were alike, for example, both have only straight lines, and the other describing how they were different, for example, one makes a fence that would keep the dog in and the other doesn't. Fourth graders worked in groups of four to find all the different shape rectangles they could make with color tiles for all the numbers from 1 to 25. (*See Figure 2–1.*)

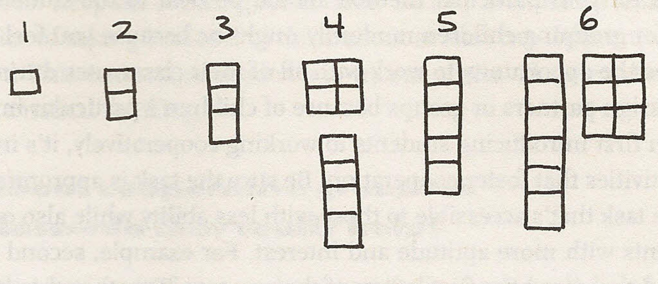
When groups are working, circulate and listen. This is an effective way to learn about how children are thinking. Join groups that seem to be stuck and offer help as needed and probe thinking when you see the opportunity. Have a class discussion afterward so that groups can hear about how other groups worked and what they discovered.

20 How can I decide whether the best approach for a lesson is for students to work individually, with partners, in small groups, or as a whole class?

Following are some simple guidelines for deciding which organization is most appropriate in a situation. If you're giving the students information that you want everyone to know about, or if one of the students is offering ideas that are valuable for the entire class to hear, then whole-class instruction makes best sense. There's no point in putting children in groups and then going around to deliver the same information to each group. It's more efficient for all of them to hear information at the same time, and this guarantees that they all receive the same version of the message.

But even when students receive the same message, they don't necessarily hear it in the same way. To process information, it helps students to have a chance to talk about it. In whole-class discussions, only one person talks at a time. In small groups, however, more of the students can talk at the same time and, therefore, more have the chance to be actively involved at the same time. This more widespread involvement is also useful when you want students to engage in an investigation or solve a problem.

However, when you want to know about what individual students understand and can do, then individual work makes best sense. At these times, ask students



2–1. Students use color tiles to build all the possible shape rectangles for different numbers.

not to talk to one another but to do their best to show you what they do and don't understand.

It's good to vary the classroom structure to involve students in all three types of situations; a variety makes instruction more engaging for the students. Also, for some lessons, you'll want a mix of groupings. For example, you might begin in a whole-class setting. You might ask second graders to estimate the number of cubes in a jar and then, as the whole class watches, empty the jar one by one and count to determine that there are 32 cubes. Then pose a problem: *If we put the cubes into groups of five each, how many groups will there be?* Have students discuss the problem in groups or with a partner. After a few minutes, call the class back to attention and discuss their ideas. Then actually put the cubes into groups of five, count the groups, and then count the cubes by 5s—5, 10, 15, 20, 25, 30, and two more makes 32. Repeat, this time asking groups to figure out how many groups there would be if you put the cubes into groups of two each. Then rearrange the cubes, count the groups, and count the cubes by 2s—2, 4, 6, 8, 10, and so on. Finally, for an individual assignment, ask students to figure out how many groups there would be if you grouped the cubes into 10s.

21 After I give my class directions for an activity, it seems that there's a flurry of hands from students who want assistance. What can I do about this?

No matter how clear you think your directions are, it always seems that some students are confused and not sure what to do. This problem exists in all subject areas, not just mathematics. We have three suggestions.

One is to have students describe what to do before you set your class loose on any assignment. Ask, "Who can give the directions in your own words?" or, "Who can say aloud what you're supposed to do now?" Urge the others to listen and be sure that they agree. Have at least two students offer their versions. After each, ask if anyone has something to change or add.

Another idea, when suitable for the particular assignment you give, is to tell students to check with a partner or the others at their table first before raising a hand. If this doesn't help, then both students or all members of a group should raise a hand to signal to you that they all have some confusion.

One more suggestion is to bring a kitchen timer to school. After giving directions and having some students repeat them, tell the class that you're setting the time for five minutes and that they need to work on their own at least until they hear the "bing." Then, if they haven't resolved their problem, you'll be happy to come and help. This system can encourage students to be more independent and self-reliant.

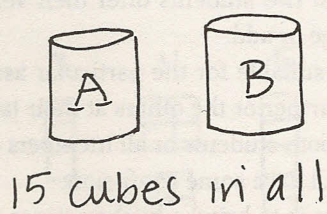
22 Some students always finish assignments quickly. What should I have them do?

This is a typical situation in class, and the best way to prepare is to anticipate it. Have an option ready for those students who learn and work more quickly. Try to make the option an extension that relates to the work at hand but offers a further challenge. When possible, offer an option that the students who finished first can work on together.

A caution: When situations like these occur, it doesn't seem right merely to assign additional similar problems to students who finish quickly. Most likely, they're the students who understand best and probably don't need more practice. That's why the extra work you give them should challenge them to extend their knowledge. (Of course, this isn't always the case, so be sure to take a careful look at papers to see if a student's work needs correction or improvement.)

For example, second graders were shown two containers, each with some cubes in it. They couldn't see how many cubes were in each, but they were told how many cubes there were in the two containers altogether. When they were first introduced to the activity, fewer than 10 cubes were used, and the children learned to record addition combinations for the possibilities. Next, as an individual assignment, they were to figure out how many cubes could possibly be in each container when there were 15 cubes in all. (*See Figure 2–2.*)

As usual, Andrew finished first and brought up his paper. A talented math student, he had approached the problem in an orderly way and was sure he had found them all. He had. Becky, Mario, and Celia also finished quickly. They didn't approach the problem in the orderly way Andrew did, but they had found most of the possibilities and felt they were done. The others were still working. The children who finished more quickly were given the challenge of figuring out the combinations for the 15 cubes if there were three containers instead of two.



2–2. What are the possible combinations of cubes that could be in the two containers?

Older students were given a similar sort of problem. Their assignment was to figure out how many red, blue, and yellow cubes might be in a container, using a set of three clues.

Clue 1: *There are fewer than 25 cubes.*

Clue 2: *Two colors have the same number of cubes.*

Clue 3: *There are twice as many blue cubes as red cubes.*

The problem had multiple solutions, and their task was to find as many of them as they could. Katia quickly found four solutions; Peter and Jaime each found six; Hattie found eight; the rest of the students were continuing to work. As an extension, students who finished first were asked to write a fourth clue that would narrow the possibilities to one correct solution.

For some assignments, it's possible to ask students who finish quickly to sit in a small group, compare their papers, and see if they all agree. If there are differences, they should try to figure out why. When they all agree, they should revise their papers and let you know. This strategy worked in both situations above. But it won't always be appropriate. Sometimes the assignment doesn't lend itself to this sort of collaboration, or you may prefer that the students tackle an extension individually.

As an organizational issue, it helps to decide ahead of time whether it's best to have written directions for the extension, if you'd prefer to explain it to children as they complete their work, or if you plan to explain it to one child who will then tell the others. Thinking through details like these will help any lesson go more smoothly.

23 I spend an hour planning a classroom activity or assignment that takes my students five minutes to complete. How can I fix this picture?

Keep in mind the "input/output" rule: A teacher's input time for planning and preparing a lesson should be less than the students' output time. This means that it should take you less time to get ready for a lesson—thinking about the mathematics, planning the instruction, gathering the materials, and preparing an assignment and extensions—than the class time required for students to engage with the lesson, by trying an activity, having a discussion, working in small groups, and completing an assignment.

We know that planning and preparing are time-consuming, especially when content is complex and perhaps new to you. Have you heard the saying that you always know something best when you've taught it? That's because teaching calls for analyzing content and then making plans for how to present it logically and effectively to students. When you analyze and plan, you think about the ideas, turn them around, look at them from different perspectives, and consider alternatives for learning activities. In fact, you engage with all of the kinds of mental activities that are essential for really learning something new—the very mental activities that you'd like your students to experience.

Be careful not to synthesize your preparation time into a tidy and tight presentation. Learning is a process that's generally more messy than orderly and calls for trying out ideas, following leads, and revising thinking. For this reason, your students need at least the same amount of time to engage with the ideas and explore them as you did in preparation. And then, of course, they need time to practice, cement, and extend their understanding. In your lessons, don't rush for closure by the end of each math period; let lessons extend beyond a day's experience when needed. Push for depthful involvement and allow the time for it.

24 It seems important to be organized and have all the details of a lesson planned beforehand, but I feel that I'm doing too much planning. How can I change this?

Let's look at an example. Suppose you want your students to operate a make-believe store in order to practice dealing with money. To prepare, you might first select a variety of classroom items to "sell." Then you might decide how much each item should cost, write the prices on stickers or index cards, and attach them to the items. You might sort the items and figure out how best to set them out. If you want each child to have the same amount of money, you could count out the same collection of coins for each student in individual zip-top plastic baggies. Whew! The idea of applying money concepts to running a store is terrific, but you would need to spend hours just to get it ready.

Think. How can you plan this experience so that the children do the bulk of the work as well as the bulk of the learning? While the practice with money will be a valuable experience for the students, other learning can occur if you think differently about organizing the activity. First discuss with children what makes a store successful. Ask them to think about items that a classroom store should sell. Perhaps they might take some opinion polls and get involved in collecting, graphing, and interpreting data about their classmates' preferences. Once items are identified, engage the class in a discussion about how they might be sorted into "departments" in their store. Have the children write prices on stickers or index cards to label each item, which gives them practice representing money correctly. Give each child the same amount of spending money, but give it to them in the greatest denomination that they are familiar with so they will have to engage immediately in making change.

In the first scenario, you do all of the problem solving needed to establish a store, limiting the children's learning to their work with money. By having children identify, price, and sort the items before they buy or sell them, you give students the chance to apply an assortment of skills. But more importantly, they are actively involved in the planning and execution of the lesson. Because they are constructing it, they have ownership, an important ingredient for motivating their participation.