POSSIBLE/ IMPOSSIBLE

GEOMETRY • MEASUREMENT • LOGIC

- Polygons
- Properties of geometric figures

Getting Ready

What You'll Need

Geoboards, 1 per child

Rubber bands

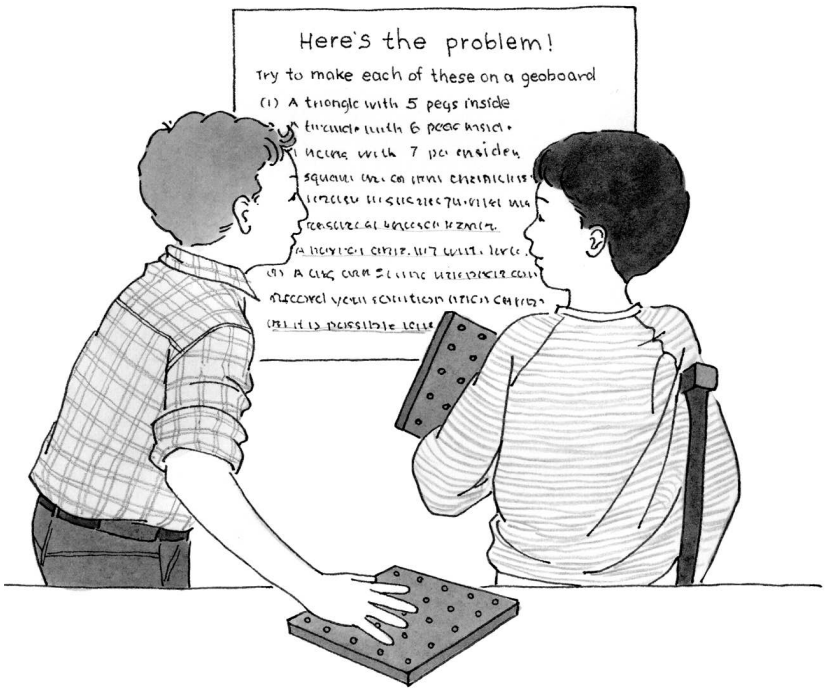
Geodot paper, pages 90–91

Overhead Geoboard and/or geodot paper transparency (optional)

Overview

Children try to make Geoboard polygons that fit given descriptions. In this activity, children have the opportunity to:

- explore attributes of different polygons
- use mathematical reasoning to determine whether or not it is possible to make a shape having a particular set of attributes
- use geometric language to describe polygons and their attributes

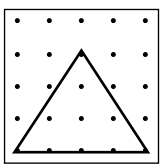
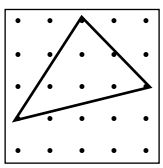
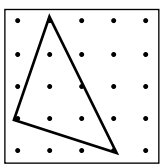


The Activity

You may want to review these terms before doing the lesson: obtuse angle, right angle, congruent angles, congruent sides, parallel sides, isosceles triangle, hexagon, convex polygon, *and* concave polygon.

Introducing

- Ask children to make a triangle that has 4 interior pegs on their Geoboards.
- Have children hold up their Geoboards. Ask them to discuss how their triangles are alike and how they are different.

On Their Own

What kinds of shapes can you make on a Geoboard?

- Work with a group to decide if it is possible or impossible to make a Geoboard shape that fits each of the descriptions below.
  - If it is possible to make a shape, record it on geodot paper. Then see if you can find and record more shapes that fit that description.
  - If it is impossible to make a shape, be ready to explain why.

Possible or Impossible?

1. A triangle can have 5 interior pegs.
2. A triangle can have 6 interior pegs.
3. A triangle can have 7 interior pegs.
4. An isosceles triangle can have one obtuse angle.
5. An isosceles triangle can have one right angle.
6. An isosceles triangle can have no angles congruent.
7. A square can have sides longer than 2 units but shorter than 3 units.
8. A square can have an area of 6 square units.
9. A hexagon can have no parallel sides.
10. A hexagon can have 3 parallel sides.
11. A hexagon can have no congruent sides.
12. A hexagon can have all sides congruent.

The Bigger Picture

Thinking and Sharing

Create 12 columns with headings corresponding to each description. Have volunteers post their solutions in the appropriate columns. For each description that they determined to be "impossible to make," have children discuss why they reached that decision. If disagreements occur, allow time for children to work together to prove or disprove their points.

Use prompts such as these to promote class discussion:

- What discoveries did you make while searching for solutions?
- Which descriptions were the easiest to make shapes for? Which were the most difficult? Why?
- How did you decide when it was impossible to find a solution?
- Do you think that some of the shapes that you labeled "impossible to make" would be possible to draw on plain paper? Explain.

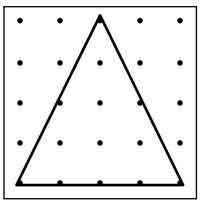
Writing

Ask children to choose some of the "impossible" descriptions and tell how to change them in order to make them "possible."

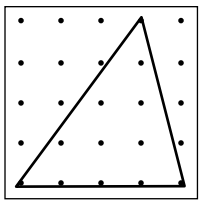
Teacher Talk

Where's the Mathematics?

The first six descriptions focus on triangles and some of their Geoboard properties. Here are "possible" triangles for the first two. Children's triangles may be congruent to these but in different orientations.



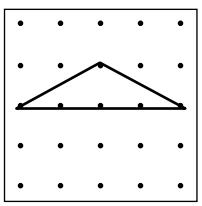
a triangle with 5 interior pegs



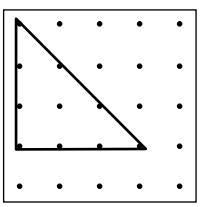
a triangle with 6 interior pegs

Some children may use trial and error to find out that it is impossible to make a triangle with 7 interior pegs (third description). Others may study the results for the first two statements and realize the impossibility of locating the third vertex so that the triangle would cover any more pegs. Thus, they come to the conclusion that the maximum number of interior pegs for a Geoboard triangle is 6.

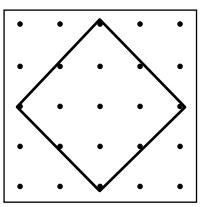
Below are some of the possible triangles for the fourth and fifth descriptions. When children compare all their isosceles triangles, they may see that isosceles triangles, in addition to having at least two sides that are congruent, always have at least two acute angles that are congruent. This fact makes the sixth description, isosceles triangle with no angles congruent, impossible.



an isosceles triangle with 1 obtuse angle



an isosceles triangle with 1 right angle



sides longer than 2 units, but shorter than 3 units

To find a square with sides longer than 2 units but shorter than 3 units, children have to realize that the diagonal distance between two pegs is greater than the horizontal or vertical distance between two pegs. Children who are not sure how to compare such distances may want to use rulers.

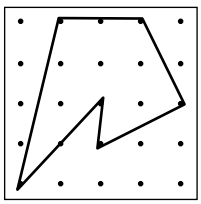
It is impossible to make a Geoboard square with an area of 6 square units. One way children may convince themselves of this is to make a rectangle

Extending the Activity

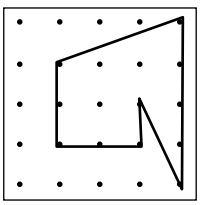
Have children work with their groups to write descriptions of polygons (some possible, some impossible) using other attributes such as symmetry, angle size, and area measure. Have them exchange lists with other groups, and find out which are possible.

with an area of 6 square units and try to adjust the side lengths to form a square with the same area.

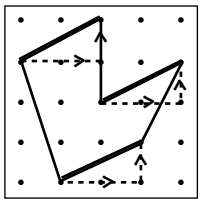
When children attempt to make hexagons with sides that are parallel, they may produce shapes that are concave, or dented, such as the ones below. In the middle hexagon, the parallel sides are easily identifiable because they are built along vertical lines of pegs. When the parallel sides are not built along vertical or horizontal lines of pegs, children may need ways to convince themselves that the sides are really parallel. One justification might be that if the sides were extended to form lines, they would never meet. Another might be that the parallel sides rise or fall at the same rate. For example, in the third hexagon below, each of the three parallel sides is formed by starting at one peg and then moving 2 pegs to the right and 1 peg upward.



a hexagon with no parallel sides

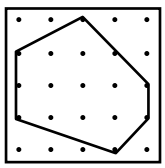


a hexagon with 3 parallel sides

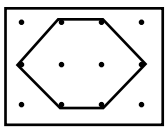


a hexagon with 3 parallel sides

Children may be surprised to see that though the eleventh description is possible, the twelfth is impossible.



a hexagon with no congruent sides



the diagonal distance between pegs is not the same as the horizontal distance between pegs

Children should realize that some shapes that are impossible to make on the Geoboard can be drawn on plain paper. For example, many different-sized squares with sides between 2 and 3 units can be drawn on paper, as can a hexagon with all congruent sides. Children's explanations as to why certain shapes cannot be made on the Geoboard may reflect the depth of their understanding about geometric concepts and the limitations of the Geoboard.